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AUGUST 16, 1961

DOWN-RANGE ANTI-MISSILE MEASUREMENT PROGRAM

DAMP TECHNICAL MONOGRAPH
NO. 61-1

DIFFERENTIAL CORRECTION TECHNIQUE FOR DETERMINATION OF THE DAMP SHIP LOCATION

By R. D. Bachinsky
B. M. Wolf

THIS RESEARCH PROGRAM IS A PART OF PROJECT DEFENDER SPONSORED BY THE **ADVANCED RESEARCH PROJECTS AGENCY** (ARPA) ARPA ORDER NO. 51

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Prepared for

ARMY ROCKET AND GUIDED MISSILE AGENCY
REDSTONE ARSENAL, ALABAMA
UNDER CONTRACT DA-36-034-ORD-3144RD

Prepared by

RADIO CORPORATION OF AMERICA
DEFENSE ELECTRONIC PRODUCTS
MISSILE AND SURFACE RADAR DIVISION
MOORESTOWN, NEW JERSEY

RCA TMK(s)®

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INTRODUCTION

The accuracies of shipboard navigational techniques, while being quite adequate for maritime purposes, are not sufficiently accurate for scientific evaluation of data recorded down-range. In order to correlate these data with data obtained in an earth-fixed reference system, it is necessary to accurately know the location of the ship relative to this earth reference system. With this information, the impact of a ballistic missile or the trajectory of a space probe can be determined in the same reference frame as that used by a fixed land-based measurement device. This paper describes a mathematical method for accurately determining the location of the DAMP ship in an earth reference system using observed tracking data on a target whose orbital elements are known.

In essence, this method relies on three basic considerations:

1. The technique requires that the ship track a target whose orbital elements are known. These elements will be considered as the standard orbital elements.
2. Assuming any ship's location and using these standard orbital elements, a set of radar parameters (range, azimuth, elevation) can be computed.
3. The actual ship's location will then be that location at which the computed radar parameters best fit the observed radar parameters.

PRESENTATION

If a target whose orbital elements are known is tracked using a noise and bias free radar, the location of the radar could be determined purely by geometric considerations. However, since this is not possible; the more sophisticated technique described here is required.

The values of the observed range, azimuth, and elevation radar data (R_o, A_o, E_o) in an earth-fixed system are assumed to be normally distributed about the values that would be observed with a noise and bias free radar (R_m, A_m, E_m). It is further assumed that these distributions are independent of each other within the time span of the observations. Based on these assumptions, the probability density of observing any combination of these parameters at any observation i, is:

$$P(R_{oi}, A_{oi}, E_{oi}) = \left[\frac{1}{\sigma_R (2\pi)^{1/2}} e^{-\frac{(R_{oi}-R_{mi})^2}{2\sigma_R^2}}\right]\left[\frac{1}{\sigma_A (2\pi)^{1/2}} e^{-\frac{(A_{oi}-A_{mi})^2}{2\sigma_A^2}}\right]\left[\frac{1}{\sigma_E (2\pi)^{1/2}} e^{-\frac{(E_{oi}-E_{mi})^2}{2\sigma_E^2}}\right]$$

$$= \frac{\exp\left\{-\frac{1}{2}\left[\frac{(R_{oi}-R_{mi})^2}{\sigma_R^2} + \frac{(A_{oi}-A_{mi})^2}{\sigma_A^2} + \frac{(E_{oi}-E_{mi})^2}{\sigma_E^2}\right]\right\}}{\sigma_R \sigma_A \sigma_E (2\pi)^{3/2}}$$

Here σ_R, σ_A, and σ_E are the standard deviations in range, azimuth, and elevation, respectively.

If the location of the ship is determined such that the values of (R_m), (A_m), and (E_m) yield a maximum probability density of having observed (R_o), (A_o), and (E_o) over all observations, then the requirements of consideration 3 stated in the Introduction are fulfilled.

This probability density for N number of observations becomes:

$$P_N(R_o, A_o, E_o) = \prod_{i=1}^{N} P(R_{o_i}, A_{o_i}, E_{o_i})$$

or,

$$P_N(R_o, A_o, E_o) = \frac{1}{(\sigma_R \sigma_A \sigma_E)^N (2\pi)^{\frac{3N}{2}}} \exp\left\{-\frac{1}{2}\sum_{i=1}^{N}\left[\frac{(R_{o_i}-R_{m_i})^2}{\sigma_R^2} + \frac{(A_{o_i}-A_{m_i})^2}{\sigma_A^2} + \frac{(E_{o_i}-E_{m_i})^2}{\sigma_E^2}\right]\right\} \tag{1}$$

The mean radar parameters are functions of the ship location and, therefore, may be individually expanded about some arbitrary location c. The linear terms of a Taylor series are used:

$$R_m = R_c + \frac{\partial R_c}{\partial \lambda}\Delta\lambda + \frac{\partial R_c}{\partial L}\Delta L$$

$$A_m = A_c + \frac{\partial A_c}{\partial \lambda}\Delta\lambda + \frac{\partial A_c}{\partial L}\Delta L$$

$$E_m = E_c + \frac{\partial E_c}{\partial \lambda}\Delta\lambda + \frac{\partial E_c}{\partial L}\Delta L$$

in which λ and L represent latitude and longitude, respectively. The "c" subscript indicates evaluation at point c and the delta expression represents the difference in position between c and the actual ship location.

Substituting these expressions for the mean radar parameters into Equation (1) yields:

$$P_N(R_o, A_o, E_o) = \frac{1}{(\sigma_R \sigma_A \sigma_E)^N (2\pi)^{\frac{3N}{2}}} \exp\left\{-\frac{1}{2}\sum_{i=1}^{N}\left[\frac{\left(R_{oi}-R_{ci}-\frac{\partial R_{ci}}{\partial \lambda}\Delta\lambda-\frac{\partial R_{ci}}{\partial L}\Delta L\right)^2}{\sigma_R^2}\right.\right.$$

$$\left.\left.+\frac{\left(A_{oi}-A_{ci}-\frac{\partial A_{ci}}{\partial \lambda}\Delta\lambda-\frac{\partial A_{ci}}{\partial L}\Delta L\right)^2}{\sigma_A^2}+\frac{\left(E_{oi}-E_{ci}-\frac{\partial E_{ci}}{\partial \lambda}\Delta\lambda-\frac{\partial E_{ci}}{\partial L}\Delta L\right)^2}{\sigma_E^2}\right]\right\}.$$

Since it is desired to maximize $P_N(R_o, A_o, E_o)$ above, then the exponent should be minimized with respect to $\Delta\lambda$ and ΔL. Therefore,

$$\frac{\partial}{\partial \Delta\lambda}\sum_{i=1}^{N}\left[\frac{\left(R_{oi}-R_{ci}-\frac{\partial R_{ci}}{\partial \lambda}\Delta\lambda-\frac{\partial R_{ci}}{\partial L}\Delta L\right)^2}{\sigma_R^2}+\frac{\left(A_{oi}-A_{ci}-\frac{\partial A_{ci}}{\partial \lambda}\Delta\lambda-\frac{\partial A_{ci}}{\partial L}\Delta L\right)^2}{\sigma_A^2}\right.$$

$$\left.+\frac{\left(E_{oi}-E_{ci}-\frac{\partial E_{ci}}{\partial \lambda}\Delta\lambda-\frac{\partial E_{ci}}{\partial L}\Delta L\right)^2}{\sigma_E^2}\right]=0$$

and

$$\frac{\partial}{\partial \Delta L}\sum_{i=1}^{N}\left[\frac{\left(R_{oi}-R_{ci}-\frac{\partial R_{ci}}{\partial \lambda}\Delta\lambda-\frac{\partial R_{ci}}{\partial L}\Delta L\right)^2}{\sigma_R^2}+\frac{\left(A_{oi}-A_{ci}-\frac{\partial A_{ci}}{\partial \lambda}\Delta\lambda-\frac{\partial A_{ci}}{\partial L}\Delta L\right)^2}{\sigma_A^2}\right.$$

$$\left.+\frac{\left(E_{oi}-E_{ci}-\frac{\partial E_{ci}}{\partial \lambda}\Delta\lambda-\frac{\partial E_{ci}}{\Delta L}\Delta L\right)^2}{\sigma_E^2}\right]=0;$$

then

$$\sum_{i=1}^{N}\left[\frac{\left(R_{o_i}-R_{c_i}-\dfrac{\partial R_{c_i}}{\partial \lambda}\Delta\lambda-\dfrac{\partial R_{c_i}}{\partial L}\Delta L\right)}{\sigma_R^2}\frac{\partial R_{c_i}}{\partial \lambda}+\frac{\left(A_{o_i}-A_{c_i}-\dfrac{\partial A_{c_i}}{\partial \lambda}\Delta\lambda-\dfrac{\partial A_{c_i}}{\partial L}\Delta L\right)}{\sigma_A^2}\frac{\partial A_{c_i}}{\partial \lambda}\right.$$

$$\left.+\frac{\left(E_{o_i}-E_{c_i}-\dfrac{\partial E_{c_i}}{\partial \lambda}\Delta\lambda-\dfrac{\partial E_{c_i}}{\partial L}\Delta L\right)}{\sigma_E^2}\frac{\partial E_{c_i}}{\partial \lambda}\right]=0$$

and

$$\sum_{i=1}^{N}\left[\frac{\left(R_{o_i}-R_{c_i}-\dfrac{\partial R_{c_i}}{\partial \lambda}\Delta\lambda-\dfrac{\partial R_{c_i}}{\partial L}\Delta L\right)}{\sigma_R^2}\frac{\partial R_{c_i}}{\partial L}+\frac{\left(A_{o_i}-A_{c_i}-\dfrac{\partial A_{c_i}}{\partial \lambda}\Delta\lambda-\dfrac{\partial A_{c_i}}{\partial L}\Delta L\right)}{\sigma_A^2}\frac{\partial A_{c_i}}{\partial L}\right.$$

$$\left.+\frac{\left(E_{o_i}-E_{c_i}-\dfrac{\partial E_{c_i}}{\partial \lambda}\Delta\lambda+\dfrac{\partial E_{c_i}}{\partial L}\Delta L\right)}{\sigma_E^2}\frac{\partial E_{c_i}}{\partial L}\right]=0$$

Combining terms yields:

$$\sum_{i=1}^{N}\left[\frac{(R_{o_i}-R_{c_i})}{\sigma_R^2}\frac{\partial R_{c_i}}{\partial \lambda}+\frac{(A_{o_i}-A_{c_i})}{\sigma_A^2}\frac{\partial A_{c_i}}{\partial \lambda}+\frac{(E_{o_i}-E_{c_i})}{\sigma_E^2}\frac{\partial E_{c_i}}{\partial \lambda}\right]$$

$$-\left[\frac{\left(\dfrac{\partial R_{c_i}}{\partial \lambda}\right)^2}{\sigma_R^2}+\frac{\left(\dfrac{\partial A_{c_i}}{\partial \lambda}\right)^2}{\sigma_A^2}+\frac{\left(\dfrac{\partial E_{c_i}}{\partial \lambda}\right)^2}{\sigma_E^2}\right]\Delta\lambda$$

$$-\left[\frac{\left(\dfrac{\partial R_{c_i}}{\partial \lambda}\dfrac{\partial R_{c_i}}{\partial L}\right)}{\sigma_R^2}+\frac{\left(\dfrac{\partial A_{c_i}}{\partial \lambda}\dfrac{\partial A_{c_i}}{\partial L}\right)}{\sigma_A^2}+\frac{\left(\dfrac{\partial E_{c_i}}{\partial \lambda}\dfrac{\partial E_{c_i}}{\partial L}\right)}{\sigma_E^2}\right]\Delta L=0 \quad (2)$$

and

$$\sum_{i=1}^{N}\left[\frac{(R_{o_i}-R_{c_i})}{\sigma_R^2}\frac{\partial R_{c_i}}{\partial L}+\frac{(A_{o_i}-A_{c_i})}{\sigma_A^2}\frac{\partial A_{c_i}}{\partial L}+\frac{(E_{o_i}-E_{c_i})}{\sigma_E^2}\frac{\partial E_{c_i}}{\partial L}\right]$$

$$-\left[\frac{\left(\frac{\partial R_{c_i}}{\partial \lambda}\frac{\partial R_{c_i}}{\partial L}\right)}{\sigma_R^2}+\frac{\left(\frac{\partial A_{c_i}}{\partial \lambda}\frac{\partial A_{c_i}}{\partial L}\right)}{\sigma_A^2}+\frac{\left(\frac{\partial E_{c_i}}{\partial \lambda}\frac{\partial E_{c_i}}{\partial L}\right)}{\sigma_E^2}\right]\Delta\lambda$$

$$-\left[\frac{\left(\frac{\partial R_{c_i}}{\partial L}\right)^2}{\sigma_R^2}+\frac{\left(\frac{\partial R_{c_i}}{\partial L}\right)^2}{\sigma_A^2}+\frac{\left(\frac{\partial E_{c_i}}{\partial L}\right)^2}{\sigma_E^2}\right]\Delta L = 0 \qquad (3)$$

It is now necessary to obtain expressions for range, azimuth, and elevation in terms of the latitude and longitude in order that the partial derivitives appearing in Relations (2) and (3) may be evaluated. These expressions are given by

$$R_c = \left[(R_E+h)^2+R_E^2-2R_E(R_E+h)\left\{\cos\lambda_S\cos\lambda_T\cos(L_T-L_S)+\sin\lambda_T\sin\lambda_S\right\}\right]^{\frac{1}{2}} \qquad (4)$$

$$A_c = \cos^{-1}\left[\frac{\sin\lambda_T\cos\lambda_S-\cos\lambda_T\sin\lambda_S\cos(L_S-L_T)}{\left\{\cos^2\lambda_T\cos^2\lambda_S\sin^2(L_S-L_T)+\sin^2\lambda_S\cos^2\lambda_T+\sin^2\lambda_T\cos^2\lambda_S-2\sin\lambda_S\cos\lambda_S\sin\lambda_T\cos\lambda_T\cos(L_S-L_T)\right\}^{\frac{1}{2}}}\right] \qquad (5)$$

$$E_c = \sin^{-1}\left\{\frac{(R_E+h)(\cos\lambda_T\cos\lambda_S\cos(L_T-L_S)+\sin\lambda_T\sin\lambda_S)-R_E}{\left[(R_E+h)^2+R_E^2-2R_E(R_E+h)\left\{\cos\lambda_S\cos\lambda_T\cos(L_T-L_S)+\sin\lambda_T\sin\lambda_S\right\}\right]^{\frac{1}{2}}}\right\} \qquad (6)$$

where R_E is the radius of the earth, h is the altitude of the target, λ_S and L_S are the latitude and longitude of the ship at location c, λ_T and L_T are the latitude and longitude of the target.

The desired partial derivatives are now,

$$\frac{\partial R_c}{\partial \lambda_S} = \frac{R_E(R_E + h)\left[\cos\lambda_T \sin\lambda_S \cos(L_S - L_T) - \sin\lambda_T \cos\lambda_S\right]}{R_c}$$

$$\frac{\partial R_c}{\partial L_S} = \frac{R_E(R_E + h) \cos\lambda_T \cos\lambda_S \sin(L_S - L_T)}{R_c}$$

$$\frac{\partial E_c}{\partial \lambda_S} = [1 - u^2]^{-\frac{1}{2}} \frac{\partial u}{\partial \lambda_S}$$

in which

$$u = \frac{(R_E + h)\left[\cos\lambda_T \cos\lambda_S \cos(L_S - L_T) + \sin\lambda_T \sin\lambda_S\right] - R_E}{R_c}$$

and

$$\frac{\partial u}{\partial \lambda_S} = \left\{\frac{(R_E + h)\left[\cos\lambda_S \sin\lambda_T - \sin\lambda_S \cos\lambda_T \cos(L_S - L_T)\right]}{R_c}\right\} - \left\{\frac{u}{R_c} \frac{\partial R_c}{\partial \lambda_S}\right\}$$

$$\frac{\partial E_c}{\partial L_S} = [1 - u^2]^{-\frac{1}{2}} \frac{\partial u}{\partial L_S}$$

in which

$$\frac{\partial u}{\partial L_S} = \left\{\frac{-(R_E + h) \cos\lambda_T \cos\lambda_S \sin(L_S - L_T)}{R_c} - \frac{u}{R_c} \frac{\partial R_c}{\partial L_S}\right\}$$

$$\frac{\partial A_c}{\partial \lambda_S} = (-1)^n (1 - V^2)^{-\frac{1}{2}} \frac{\partial V}{\partial \lambda_S} \quad \text{where} \quad \begin{array}{l} n = 1 \quad \text{for} \quad 0 \le V < \pi \\ n = 2 \quad \text{for} \quad \pi \le V < 2\pi \end{array}$$

where

$$V = \frac{(R_E + h)\left[\sin\lambda_T \cos\lambda_S - \cos\lambda_T \sin\lambda_S \cos(L_S - L_T)\right]}{\omega}$$

and

$$\omega = (R_E + h)\Big[\sin^2\lambda_S \cos^2\lambda_T + \sin^2\lambda_T \cos^2\lambda_S + \cos^2\lambda_T \cos^2\lambda_S \sin^2(L_S - L_T)$$

$$- 2 \sin\lambda_S \cos\lambda_T \sin\lambda_T \cos\lambda_S \cos(L_S - L_T)\Big]^{\frac{1}{2}}.$$

Also,

$$\frac{\partial V}{\partial \lambda_S} = \frac{(R_E + h)\left[-\sin\lambda_S \sin\lambda_T - \cos\lambda_T \cos\lambda_S \cos(L_S - L_T)\right]}{\omega}$$

$$- \frac{V}{\omega^2}\Big\{\sin\lambda_S \cos\lambda_S\left[\cos^2\lambda_T \cos^2(L_S - L_T) - \sin^2\lambda_T\right]$$

$$- \sin\lambda_T \cos\lambda_T(\cos^2\lambda_S - \sin^2\lambda_S)\cos(L_S - L_T)\Big\}(R_E + h)^2.$$

$$\frac{\partial A_c}{\partial L_S} = (-1)^n (1 - V^2)^{-\frac{1}{2}} \frac{\partial V}{\partial L_S} \quad \text{where} \quad \begin{array}{l} n = 1 \quad \text{for} \quad 0 \le V < \pi \\ n = 2 \quad \text{for} \quad \pi \le V < 2\pi \end{array}$$

in which

$$\frac{\partial V}{\partial L_S} = \frac{(R_E + h)\left[\cos\lambda_T \sin\lambda_S \sin(L_S - L_T)\right]}{\omega}$$

$$- \frac{V}{\omega^2}\Big\{\cos^2\lambda_T \cos^2\lambda_S \sin(L_S - L_T)\cos(L_S - L_T)$$

$$+ \sin\lambda_T \cos\lambda_T \sin\lambda_S \cos\lambda_S \sin(L_S - L_T)\Big\}(R_E + h)^2.$$

These expressions for the partial derivatives may now be substituted into Relations (2) and (3) and a solution for ΔL and $\Delta\lambda$ obtained based on all observations. However, these values represent a correction to the assumed location and may still contain error. This error results from using only the linear terms of the Taylor expansion.

Successive iterations using these corrected values as the new locations yields a sequence of values for ΔL and $\Delta\lambda$ which converge toward zero.

A possibility exists that a non-convergent sequence may result. However, in these investigations, which assumed rather large errors in initial location, this did not occur. Related investigations using these techniques have shown that under these conditions the simple expedient of selecting a different location produced convergent sequences.

A series of simulated tests were performed using orbital elements from an actual ballistic missile trajectory to determine the feasibility of this method. The simulations involved the following basic procedure.

1. A location was chosen from which a noise-free set of radar parameters were computed.
2. These parameters were then contaminated with gaussian noise of known standard deviation to simulated observed radar parameters.
3. Using these parameters and assuming an arbitrary ship's location, in conjunction with the known orbital elements, the method was applied to determine the simulated actual location.

It should be noted that one of the inputs is the standard deviation of the noise, which in an actual mission may not be known. In order to be realistic, the input values of standard deviations were selected to be different from those used in establishing the simulated observed radar parameters.

The values of the input standard deviations assigned to each of the radar parameters (range, azimuth, and elevation) are measures of their confidence level: i.e., if one of the parameters is heavily contaminated with noise, its confidence level is low.

One of the outputs resulting from performing these calculations is the actual standard deviation of the observed data about the mean. This is a measure of the validity of the selected standard deviations and indicates whether or not the confidence levels have been wrongly emphasized.

RESULTS AND CONCLUSIONS

A table listing the conditions and results of the simulation program follows. True ship's location was selected on the basis of a "worst case" situation where the ship lies very close to the plane of the trajectory.

In the first simulation listed, the input values of the standard deviation were the same as the values used to contaminate the radar parameters. Even with the rather large errors in assumed ship's location, the technique indicated true location to slightly more than six feet.

The next four simulations tested the method for varying degrees of assumed noise contamination. Note that regardless of the amount of error in the assumed noise, or whether it exceeds or falls below the true standard deviation, the location is determined to the same degree of accuracy, 30 feet in latitude and 241 feet in longitude. It must be kept in mind that the technique, in addition to correcting for ship's location, yields corrected information on standard deviation that may be re-inserted to provide a ship's location with an accuracy commensurate with that determined in the first simulation.

The last simulation not only introduced noise, but also an amount of bias equal to that of the noise. Under these conditions, location in latitude was determined to within 1471 feet and in longitude to within 1050 feet. These results indicate that bias errors in the radar parameters effect the accuracy of determining the position of the ship.

In any radar system there exists many sources which introduce bias errors. Knowing these sources, it is possible to extend this differential correction technique to compute and compensate for these errors. For the DAMP tracking radar the major bias errors are introduced in the determination of true north and the local vertical. However, in the above simulation these errors were not considered since this investigation was undertaken primarily to determine the feasibility of this method. The technique is currently being extended to include bias errors in true north and local vertical.

TABULATED RE

SHIP'S LOCATION DIF

CORRECTIO

TIME OF INITIAL RADAR
OBSERVATION (SECONDS AFTER MIDNIGHT): 53826.0

TRU

TIME BETWEEN OBSERVATIONS: 1.0 SECOND

TIME OF FINAL RADAR
OBSERVATION (SECONDS AFTER MIDNIGHT): 54026.00

SIMULATION NUMBER	RADAR BIAS ERRORS ASSIGNED (Degrees)		(Yards)	TRUE STANDARD DEVIATION (Degrees)		(Yards)	ASSUMED STANDARD DEVIATION (Degrees)		(Yards)	ASSUMED SHI POSITION (Degrees)	
	A	E	R	A	E	R	A	E	R	Latitude	Long
1	0	0	0	0.25	0.25	15.0	0.25	0.25	15.0	8.713972S	17.
2	0	0	.0	0.25	0.25	15.0	0.5	0.5	15.0	8.713972S	17.
3	0	0	0	0.25	0.25	15.0	2.5	2.5	15.0	8.713972S	17.
4	0	0	0	0.25	0.25	15.0	0.125	0.125	15.0	8.713972S	17.
5				0.25	0.25	15.0	25×10^6	25×10^6	15.0	8.713972S	17.
6	0.25	0.25	15.0	0.25	0.25	15.0	0.25	0.25	15.0	8.713972S	17.

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TABULATED RESULTS
SHIP'S LOCATION DIFFERENTIAL CORRECTION

326.0 TRUE SHIP'S POSITION: 7.713972° SOUTH
19.00° WEST

026.00

	ASSUMED ANDARD DEVIATION (Degrees)	(Yards)	ASSUMED SHIP'S POSITION (Degrees)		MINIMUM NUMBER OF ITERATIONS REQUIRED	FIRST ITERATION POSITION (Degrees)		LAST ITERATION POSITION (Degrees)		ERROR IN LAST ITERATION (FEET) FROM TRUE SHIP'S POSITION	
	E	R	Latitude	Longitude		Latitude	Longitude	Latitude	Longitude	Latitude	Longitude
	0.25	15.0	8.713972S	17.0W	4	8.248942S	19.41574W	7.713993S	19.00002W	6.6	6.3
	0.5	15.0	8.713972S	17.0W	4	8.249214S	19.41537W	7.713876S	18.99923W	30.4	241
	2.5	15.0	8.713972S	17.0W	4	8.249509S	19.41564W	7.713877S	18.99924W	30	241
	0.125	15.0	8.713972S	17.0W	4	8.244584S	19.41119W	7.713876S	18.99924W	30.4	241
10^6	25×10^6	15.0	8.713972S	17.0W	4	8.249529S	19.41566W	7.713874S	18.99924W	29.7	241
	0.25	15.0	8.713972S	17.0W	4	8.240465S°	19.40939W	7.709324S	18.99668W	1471	1050

TRAJECTORY ORBITAL ELEMENTS

Semi-major axis:	0.82363	Argument of Perigee:	335.4405
Time of Perigee:	51464.27	Eccentricity:	0.43272
Right Ascension:	25.27468	Inclination:	31.80759

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Missile and Surface Radar Division
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Contract DA-36-034-ORD-3144RD ARGMA-ARPA
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The feasibility of the method was investigated through a series of simulated tests using orbital elements from an actual ballistic missile trajectory. Varying degrees of noise and bias were introduced to determine the effects on the accuracy of locating the mobile radars.

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